University of Hull

Report to Analyse the Simulation of Random Growth of Cells

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**Introduction**

Computational science is the discipline within computer science that advances knowledge in various scientific fields through complex computational analysis, modelling, simulation, and numerical approximation (Plonus, 2007). It can provide innovative solutions to problems by using powerful computers to replace or complement traditional modelling and experimental methods.

This study aims to understand the different complexities that encapsulate growth simulations and how they can be applied to real-world scenarios. Building upon various random grid walk models to understand random movement, which can be further applied to random cell growth, simulating a cancer tumour. The Gompertz model will support this simulation by applying more realistic characteristics, such as time, cell capacity, and growth rate. Combining these methods shall form a basic simulation of how a tumour can grow and multiply over time in a living organism.

**Task 1.1**

The first steps to create the simulation include creating a random grid walk function. The goal for this task is for a cell to start in the middle of a one hundred by one hundred grid and move randomly: left, right, up and down one grid place for one hundred steps.

This task relies on two random number generators and a simple binary algorithm to decide at random whether the cell should move in any of the four directions. This algorithm loops one hundred times to simulate the movement for one hundred steps.

However, one issue with this algorithm is the implementation of two random number generators. As a computer cannot be biologically genuinely random, we must implement pseudo-random number generators to simulate randomness. A pseudo-random number generator is an algorithm that ‘produces a deterministic stream of seemingly random numbers’ (Reeds & Walkenhorst, 2011). A distribution type must be applied to determine the range and type of outputs to expect from the function. The two main types considered for this project are uniform and normal distribution.

A comparison of a graph

Description automatically generated with medium confidence**A comparison of a graph

Description automatically generated with medium confidence**‘Normal distribution is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data from the mean’ (Chen, 2024). For example, if a normal distribution function had a mean of 0 and a standard deviation of 1, most results would be in the range of -1 and 1, primarily around 0, and values out of this range are rarely selected in comparison. Resulting in a ‘bell curve’ when graphed, as shown in Figure 1. Normal distribution can often occur naturally. ‘For example, the average height of a human is roughly 5’ 9”’(Chen, 2024). Taller and shorter people exist but decrease in frequency within the population. Thus, normal distribution would be helpful in simulations relating to natural trends such as population height.

**Figure 1: Normal distribution graph**

Alternatively, ‘uniform distribution refers to a type of probability distribution in which all outcomes are equally as likely to occur’ (Chen, 2024). Consider a coin toss uniform distribution because the odds of getting either heads or tails are even. Uniform distribution can also be plotted on a graph, as shown in Figure 2. Ideally, the graph will be a straight horizontal line. However, this is not always the case on small sample sizes due to variance, but this is less noticeable within a large dataset.

**Figure 2: Normal distribution graph**

Concerning the problem in this task, a uniform distribution shall be implemented as a random distribution, as there is no bias towards one result, and we can ensure even possibilities for all results. Like the coin toss analogy, two uniform distribution random number generators provide a binary result of one or zero.

**A grid with numbers and letters

Description automatically generated**Once implemented, the random numbers are assigned to a direction. A simple algorithm will determine the result of the two binary values. Thus, (0 0)2 move right by one, (0 1)2 move left by one, (1 0)2 move down by one and (1 1)2 move up by one. This formula covers all random outcome possibilities and all possible directions of movement. This algorithm is iterated over one hundred times to represent one hundred steps, as shown in Figure 3.

**Figure 3: Random walk graph for 100 steps**

Figure 3 depicts the task's results, with the point moving up twenty-six times, down twenty-seven times, left nineteen times, and right twenty-eight times. As uniform distribution was implemented, the point was expected to move evenly twenty-five times in all directions. However, due to a small sample size variance, known as statistical fluctuation, the results are slightly off what was predicted, as the point moved left six times less than anticipated. However, it is expected by the ‘law of large numbers’ that a larger sample will represent a population mean (Taboga, 2021). Thus, as greater step size is applied to the algorithm, results will be expected to align with uniform distribution.

**Task 1.2**

Continuing from the previous task, the model will be progressed further. Instead of four possible directions, the cell will now have eight, including the four previous directions and all diagonals. In addition, the step count will increase from one hundred to one thousand and ten thousand steps.

To implement the new movement functionality, another uniform distribution random number generator shall be combined with the previous two generators. This will provide a three-digit binary result: (0 0 0)2 to (1 1 1)2. These digits can represent eight possible different outcomes, each representing a potential direction of travel for the cell.

Again, the algorithm is iterated through for the given number of steps: one thousand, as shown in Figure 4, and ten thousand, as shown in Figure 5.

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**Figure 4: Random walk grid for 1,000 steps**

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**Figure 5: Random walk grid for 10,000 steps**

Figures 4 and 5 both display what appears to be a truly random pattern, with the cell continuously moving in all eight directions. The two graphs have little to no relation due to their random nature. However, there are some similarities as the cell travels around the centre point of the grid, as that’s where the starting position is for both models. Thus, these sections of the grid are more perceptible to be traversed.

Figure 6 represents the percentage number of traversals for all directions in both random walk models: 1,000 and 10,000 steps shown in the above figures. Uniform distribution states that each outcome should have an equal probability of being output. Therefore, each possibility is expected to be equal. Since there are eight possibilities, the likelihood of each is one-eighth, twelve-point-five as a percentage. As discussed before, ‘the law of large numbers’ (Taboga, 2021) is proven correct by the larger dataset of ten thousand steps, as all outcomes are around the expected number of traversals. The smaller dataset of 1,000 steps has a significant outlier in the left direction, with 3.2% traversals over the predicted value. Overall, uniform distribution has proven to be the correct choice for this implementation when using a large dataset, for example, ten thousand iterations.

**Figure 6: Percentage Frequency of Traversals for all Directions in Random Walk Model for 1,000 steps and 10,000 steps**

**Task 2.1**

The Gompertz model is a mathematical formula which has been ‘frequently used to describe the growth of animals and plants, as well as the number or volume of bacteria and cancer cells’ which slow over time (Tjørve KMC & Tjørve E, 2017). For this task, the model shall be used to simulate the growth of a tumour over time.

Figure 7 displays a graph depicting the model's results over one thousand and two hundred days. This graph represents an initial cell size, a condition set to 109, growing over time at a growth rate of 0.006 until reaching the environment's maximum carrying capacity, set to 1013.

The plot illustrates how the number of tumour cells evolves to produce a sigmoid-like curve. In the early phase of development, from days zero to two hundred, the number of cells hardly grows and starts to increase exponentially. This happens because, at the start, there are fewer cells from which to multiply. The cells continue to multiply over the next four hundred days due to few resource limitations, such as available space to grow and a rise in tumour cells. From day six hundred onward, the rate of growth begins to slow. This is due to an increase in competition for resources. As the tumour size approaches carrying capacity, the model reaches a steady state around day one thousand, and growth nearly stops by the final days.

A graph with a blue line

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**Figure 7: Graph depicting the simulation of tumour growth using the Gompertz Model over 1200 days**

Figure 8 depicts a graph that shows the time, in days, it takes for the model to reach a steady state. The steady state is when the graph begins to stop changing significantly over time as it reaches the carrying capacity. This can be calculated in several ways, such as computing the significance between step size and stopping when the difference is negligible, visually observing a plateau of a curve or calculating the tumour percentage size for the given capacity. It is clear from Figure 8 that there is a direct positive correlation between time to reach a steady state and maximum carrying capacity.

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**Figure 8: Graph to depict the time taken for the model to reach a steady state at differing carrying capacities**

**Task 2.2**

Using the previous tasks, we can simulate the random growth of a cancer tumour cell on the grid in any direction. The Gompertz model is used to implement realism and constraints to the model using predefined hyperparameters. Once the model reaches the steady state value, the tumour can grow randomly on the grid to a cell that has not yet been infected, allowing the cells to multiply as the model develops.

A grid with blue lines

Description automatically generated with medium confidenceFigure 9 displays the model's growth over time, with each diagram increasing by 20 iterations of the model. Step 0 represents the initial state of the tumour, where a single cell occupies 0.01% of the grid. Steps 20 and 40 indicate the tumour has started to grow, forming a small cluster of points spreading from the origin, covering roughly 5% of the grid. The boundaries begin irregular, reflecting natural radial growth. The following steps show the tumour has continued to grow significantly, spreading across large portions of the grid. Finally, the tumour has reached a substantial size, still in the exponential growth phase, spreading to roughly 30-35% of the grid. This model clearly demonstrates the natural and non-linear random fashion of cancer tumour growth applying the Gompertz model to simulate realistic growth over an extended period.

**Figure 9: Simulation of cell growth over several iterations**

Like the growth-time graph shown in Figure 7, the cells follow the same pattern: they start to grow slowly as there are few cells to multiply by, then grow exponentially as there are more cells to multiply, and there is not yet a contest for resources. Following this pattern and the previous graph, we can infer that growth will slow as free space is limited and the model reaches a steady state as the carrying capacity is approached. This is supported by Figure 10, which demonstrates the differing time for the model to reach a steady state depending on the grid size. There is a direct correlation between maximum capacity and time to reach a steady state. As the grid increases in size, the step size almost doubles to reach a steady state for each iteration of the maximum capacity.

**Figure 10: Graph to show the steps taken to reach a steady state with differing maximum capacities**

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